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INTRODUCTION

The Institute for Computer Applications in Science and Engineering (ICASE) is operated at the Langley Research Center (LaRC) of NASA by the Universities Space Research Association (USRA) under a contract with the Center. USRA is a nonprofit consortium of major U. S. colleges and universities.

The Institute conducts unclassified basic research in applied mathematics, numerical analysis, and computer science in order to extend and improve problem-solving capabilities in science and engineering, particularly in aeronautics and space.

ICASE has a small permanent staff. Research is conducted primarily by visiting scientists from universities and from industry, who have resident appointments for limited periods of time, and by consultants. Members of NASA's research staff also may be residents at ICASE for limited periods.

The major categories of the current ICASE research program are:

- Numerical methods, with particular emphasis on the development and analysis of basic numerical algorithms;
- Control and parameter identification problems, with emphasis on effective numerical methods;
- Computational problems in engineering and the physical sciences, particularly fluid dynamics, acoustics, and structural analysis;
- Computer systems and software, especially vector and parallel computers.

ICASE reports are considered to be primarily preprints of manuscripts that have been submitted to appropriate research journals or that are to appear in conference proceedings. A list of these reports for the period October 2, 1987 through March 31, 1988 is given in the Reports and Abstracts section which follows a brief description of the research in progress.

Presently, ICASE is operated at NASA Langley Research Center, Hampton, VA, under the National Aeronautics and Space Administration, NASA Contract No. NAS1-18107. In the past, support has been provided by NASA Contracts No. NAS1-17070, NAS1-17130, NAS1-15810, NAS1-16394, NAS1-14101, and NAS1-14472.



RESEARCH IN PROGRESS

Saul Abarbanel

Research was continued in two directions: 1. Compact high order schemes, which were derived for the Euler equations in two and three space dimensions, were shown to be effective in describing steady-states. Work has now progressed to preserve the high spatial accuracy also during the temporal evolution. This introduces implicitness into the algorithm – but it is very mild. 2. In the case of the Navier-Stokes equations, a similar approach of increasing the spatial accuracy by removing the dispersion due to advection yields also a compact scheme in the sense that the stencil is now 5×5 which is the same stencil required for the second order conservation algorithms.

H. T. Banks and F. Kojima

We are continuing our investigations on inverse problems arising in thermal testing of materials in space structures. This problem is related to the detection of structural flaws, such as broken fibers or cracks in fiber reinforced composite materials. Our attempt on this problem is directed toward the identification of the geometrical shape of boundaries for a 2-D diffusion system under boundary observations. By using a coordinate transformation, the parabolic PDE defined on an unknown spatially varying domain is converted into the same type PDE with unknown coefficients on a fixed domain. As a result, our problem can be placed within the theoretical framework of the so-called "compactness method" for parameter estimation in PDE models. The practical utility of our proposed algorithm is supported through numerous simulation experiments. For the case where the unknown boundary shape is represented by a simple function of one variable, we investigated the robustness of the proposed algorithm for noisy observed data, and we also checked the sensitivity of it with respect to the number of sensors. We are currently pursuing the development of identification methods for the "irregular" boundary structures, such as identification of material shapes with holes.

H. T. Banks and I. G. Rosen

We have extended the approximation theory we developed for the identification of nonlinear distributed parameter systems to inverse problems involving nonautonomous nonlinear systems. The approach we took was operator theoretic in nature and yielded a rather general and complete convergence theory for a wide class of problems. The first phase of our numerical study involving the identification of a quasi-linear model for heat conduction/mass transfer was carried out on a supercomputer. We are at present preparing

to begin the second phase of the computational component of our effort which will involve the application of our theory to a broader class of nonlinear systems.

Alvin Bayliss

We are continuing our numerical study of spatial instability in a supersonic boundary layer. This work is in collaboration with L. Maestrello (Transonic Aerodynamics Division, LaRC) and R. Krishnan (Vigyan Research Associates). The problem is being studied by numerically solving the three-dimensional unsteady Navier-Stokes equations using a fourth order accurate finite difference scheme. We impose a disturbance at the inflow boundary which is a superposition of a mean state and an eigenfunction obtained from linear stability theory. We have validated our code by comparing computed growth rates in the linear regime with those predicted from linear stability theory. We have obtained good agreement for several frequencies. In addition we have computed nonlinear distortion in the solution as the wave grows downstream. At present we are computing more strongly nonlinear disturbances. An ICASE report is in preparation on this work.

Dennis W. Brewer

Research is continuing on parameter estimation problems associated with linear evolution equations in infinite-dimensional spaces. A general algorithm based on quasi-linearization has been established along with its local convergence properties. The algorithm has been numerically tested on linear delay-differential equations. Numerical experiments indicate that the method converges rapidly when used to identify one or two unknown delays. Greater difficulties are encountered in coefficient identification because the solution may be insensitive to certain changes in the coefficients. Numerical experiments indicate that these problems may be reduced by adding non-homogeneous terms. Future research will involve continuing numerical experimentation and improvements in the theory to accommodate a wider class of problems.

Dennis W. Brewer and J. Steven Gibson

We are continuing our efforts to develop robust software routines for identifying parameters in robotic manipulators. The programs use numerical integration of nonlinear differential equation models and nonlinear optimization algorithms. This work is in conjunction with J. Pennington, F. Harrison, and D. Soloway (Information Systems Division, LaRC) who have provided experimental data. Our previous research indicates a need to model the integrated electro-mechanical system to reduce parameter sensitivity. We are currently developing and testing models for two-link motions of the manipulator. Future research will involve the simulation of multiple-link motions and extensions of our model to manipulators with flexible joints and novel link geometries now being designed for LaRC.

John Burns

We are continuing our effort on the development of approximation schemes for modeling and control of distributed parameter systems. We have investigated several schemes for solving the linear quadratic regulator problem for systems governed by hyperbolic partial-functional differential equations. The early schemes were based on finite element/averaging schemes with uniform mesh. The recent improvements by Fabiano are being incorporated in our newest codes.

We are also addressing some fundamental issues concerning the impact of approximation on system properties. We have shown that the choice of approximation scheme plays a tremendous part in determining the conditioning of the finite dimensional control problem. Various finite difference and finite element schemes are being investigated to determine which are more appropriate for use in control design. We are currently concentrating on diffusion and wave equations. However, we plan to begin a study of simple control problems in the area of fluid dynamics.

Richard Carter

Due to their fast local convergence properties, Newton and Newton-like methods are popular techniques for obtaining numerical solutions to nonlinear equations and nonlinear optimization problems. With the addition of trust region strategies, these algorithms are also remarkably robust. Hence, extending trust region methods to more general settings than the original formulations allow has been an active topic in the optimization community in recent years.

A variety of topics are being pursued relating to trust region methods, focusing on their application to optimization/simulation problems. In many instances it is prohibitively expensive to compute high accuracy function and gradient values for these problems since each such evaluation involves the numerical solution of one or more differential equation. As computational costs can increase geometrically with increasing accuracy, trust region methods that allow inexact function and gradient values (and which specify practical tests for admissible error levels) could lead to dramatic computational savings whenever error estimates are available.

We have previously established global convergence results for a class of methods that enforce a simple relative error bound on the gradient approximations. We extend this theory to include inexact function values by including a less elegant (but still practical) condition on the error in the difference between successive function evaluations. Results of numerical tests using a large number of synthetic test problems have been very promising. If these problems are typical and if computational costs per gradient evaluation in a given application increase geometrically as relative accuracy is improved, then a properly selected

relative error limit can reduce computational expense by several orders of magnitude. Moreover, the expense curve is very flat near the optimal relative error limit, simplifying the task of choosing this parameter. Further numerical studies are being performed using problems supplied by Tom Banks and Fumio Kojima.

A generalization of the gradient relative error condition that depends on the angle between the error and the gradient approximation has also been established. This condition is automatically satisfied if the error is orthogonal to the gradient approximation, hence the generalization may prove useful for gradient approximations directly computed by Galerkin methods. The least-change projections used in secant methods are also being investigated in this context. All of these results are to be presented at the SIAM Annual Meeting in Minneapolis, July 1988.

In collaboration with E. Armstrong and S. Joshi (Guidance and Control Division, LaRC), the unification of optimization with integrated design projects is being considered. Trust region algorithms seem quite attractive in this context due to their robust performance, yet further developments are needed to address the diversity inherent in typical projects. Topics that must be ultimately addressed include multiple objectives, nonlinear constraints, infinite and semi-infinite programming, and problems with large numbers of isolated local minimizers. As a pilot project, we are investigating a particular nonlinearly constrained nonconvex optimization problem associated with robust control of the SCOLE laboratory experiment.

Tom Crockett

Work is continuing on file concepts for parallel processing. A parallel file is defined as one which supports concurrent access by multiple processes and also utilizes parallelism in the I/O system to improve performance. The role of parallel files in MIMD computer systems was examined, and the need for both conventional and parallel access to these files was identified. Specifications for a UNIX-like system call interface to support striped files were developed, and it was determined that the striped organization could support both high-speed access by a single process, as well as self-scheduled access by multiple processes. Implementation of the system call specifications on the FLEX/32 computer is in progress. This implementation will be used to conduct experiments with up to eight parallel disks, and various factors affecting performance will be examined.

A presentation on systems issues related to the commercialization of the Navier-Stokes Computer was given at Concurrent Computer Corporation. Various topics of concern were discussed, including programmability, price/performance, I/O, and operating system interfaces. Perspectives were also given on a number of open architectural design issues which Concurrent is trying to resolve. Considerable interest was found in the graphical editor

concept, developed previously in collaboration with Sherry Tomboulian and David Middleton.

Considerable technical support was given to Langley's Computational Structural Mechanics group and Flexible Computer Corporation during installation and checkout of the parallel disk subsystem on the FLEX/32 computer. Assistance is also being given to Mary Ann Bynum of CSM in porting that group's testbed software to run under Flexible's MMOS operating system.

Naomi H. Decker

Work continues on the design, analysis and implementation of a simple, practical version of the new multigrid algorithms which employ multiple coarse grids per grid level. There are certain variants of these new algorithms which look quite promising, but the coarse grid operators are different for each coarse grid at a given grid size. The theoretically optimal coarse grid operators (the "variational" or "Galerkin" operators as in standard multigrid theory) must be derived from the intergrid operators and the fine grid operator. We have shown that the coarse grid operators can be modified slightly (as in standard multigrid) and can be expressed in terms of simple recursion relations. Without these modifications, the new multiple coarse grid methods can not be competitive in overall computational cost with the standard multigrid algorithms which use a single coarse grid per grid level.

With D. Kamowitz, work continues on applying these methods to solving the linear systems arising from time-implicit discretizations of hyperbolic conservation laws. Work is also in progress to understand the potential of these methods in the fully implicit (no time stepping) discretization of the steady state Euler equations.

Naomi H. Decker and Vijay K. Naik

Algorithms based on multigrid techniques are known to be among the most efficient methods for solving a wide class of partial differential equations on sequential machines. Recently several researchers have studied the effects of parallelizing the intra-grid and inter-grid operations. Some researchers have proposed special purpose architectures that allow one to map some important problems directly onto the hardware and thus reduce the communication delay effects. A few have proposed new concepts that obtain the solution by first establishing operators which can separate the error components in the approximations in different frequency domains that can be eliminated efficiently in parallel; but none of these studies have conclusively shown that the optimality observed on the sequential machines is conserved in time and in the number of processors needed when these methods are implemented on multiprocessors, nor is there any clear understanding about the com-

munication, the synchronization, and the preprocessing costs involved. This is especially true when the problems considered are nonlinear with mixed boundary conditions—the type of problems that need to be solved on the high performance multiprocessor systems. Our effort is directed towards understanding the limitations of efficiently parallelizing the multigrid-like concepts when used in solving nonlinear equations such as the Navier-Stokes equations.

Naomi H. Decker and Shlomo Ta'asan

Although it is widely acknowledged that multigrid iterative methods lead to fast elliptic solvers, the technical nature of the multigrid analysis usually deters rigorous analysis. There exist methods which provide a means to calculate theoretical multigrid convergence rate bounds for a large class of multigrid solvers for positive definite elliptic problems. Unfortunately, the motivation and proofs of these bounds are relatively difficult. An easier, but separate calculation, based on Fourier analysis, provides exact convergence rates for the two-grid algorithm, but the exact analysis becomes too complicated to carry out for more than two or three grids.

We have been putting together simple and easily understood techniques in the interest of providing non-technical and easily obtainable convergence estimates for any number of grid levels. These techniques simplify both the calculation of convergence rate bounds and the theoretical justification that only a fixed number of iterations will solve the problem (i.e., multigrid is a fast solver). Although, as yet, these methods are strictly valid for only a few model problems and for only the most basic types of V-cycles, they provide a framework for understanding the details of the basic interaction between the coarse grid correction and the relaxation. We have derived new estimates of the convergence rates for the k-grid problems which are no more difficult to calculate than for the two-grid problem. These convergence rates are exact for two grids, and lead to the known grid-independent rates as k becomes large.

Raad Fatoohi

A set of multitasked codes for solving the Cauchy-Riemann equations, diffusion equation, and two-dimensional Navier-Stokes equations on four processors of the Cray/2 has been developed. These codes are currently being tested on dedicated time. The focus of this research is to analyze the performance of the Cray/2 using multitasking and to determine how well, or poorly, a set of algorithms would map onto the architecture. A study of the vector performance of the Cray/2 compilers in scheduling arithmetic and memory access operations is underway. In this study, basic operations, basic routines, and an application code are considered. The impact of various memory strides are also being

investigated. It is expected that this research will help in understanding the execution behavior of some algorithms on the Cray/2.

James Geer

Work is continuing on the development and study of a hybrid perturbation/Galerkin method to determine how the method might be applied to some fluid dynamic and scattering problems. In particular, we are exploring the idea of using the method to combine information contained in perturbation expansions about several different values of the perturbation parameter. Several model problems involving regular and/or singular perturbation expansions have been studied to help to better understand the method and to determine the type of problems to which the method might be applied. At present, these model problems include some linear and nonlinear two point boundary value problems with boundary layers (e.g., laminar flow in a channel with porous walls) and some simple exterior fluid flow and scattering problems involving very slowly converging perturbation series (e.g., acoustical scattering by a slender body). In addition, we are beginning to explore the combination of our method and some homotopy methods to describe fields about geometrically complicated bodies. So far, the results have been very encouraging. Work on the method itself is being done with Dr. Carl Andersen of the College of William and Mary, while possible applications are being discussed with Dr. Eddie Liu (Low-Speed Aerodynamics Division, LaRC) and Dr. Mike Hemsch (Transonic Aerodynamics Division, LaRC).

Investigations are also continuing concerning some fundamental properties (both from theoretical and computational points of view) of a class of "almost" singular integral equations of the first kind which are useful in representing solutions to certain elliptic exterior boundary value problems. These equations typically have the property that the domain of integration R is a proper subset of the domain of validity D of the equation. Special consideration is being given to the idea of analytically continuing the solution into the domain D. In fact, this has now been done for a large class of one dimensional integrals, such as those which occur in the representation of solutions involving a body of revolution. For this special class, a characterization of R in terms of certain properties of the analytic continuation of the kernel has been obtained which leads to a simple numerical procedure to determine R and helps to circumvent some of the stability problems inherent in solving integral equations of the first kind. Application of the results to several two and three dimensional problems involving slender or thin bodies are being carried out. The symbolic manipulation system MACSYMA has been used in some of the preliminary investigations.

Steve Gibson and Dennis Brewer

We have developed new adaptive identification methods for robotic manipulators and applied these methods successfully to simulated data. We have produced preliminary adaptive identification results with experimental data from a robot in the NASA Langley Automation Technology Laboratory. More mathematical, numerical and experimental research in conjunction with automation laboratory has been planned.

Steve Gibson and Gary Rosen

We have continued theoretical and numerical research on optimal discrete-time boundary control of distributed systems, with applications to control of diffusion processes and flexible space structures. We have investigated linear-quadratic-Gaussian compensator design for such systems and computed approximations to optimal control and estimator gains as examples. One purpose of the examples has been to compare model approximations with finite element approximations in controller design.

David Gottlieb

We have developed a spectral scheme which is non oscillatory and can preserve spectral accuracy. This is obtained by adding a sawtooth function to the usual spectral basis functions. The method involves using cell averaging techniques and the spectral method is applied in the stage of recovery of point values from the cell averages. An ICASE report is in preparation.

We are constructing preconditioners to the spectral matrices. We show that very simple diagonal preconditioners reduce the condition number of first derivative matrices to O(N) and of second derivative matrices to O(N*N). Also we have constructed analytically the inverses of the Chebyshev derivative matrices.

Chester E. Grosch

The study, in collaboration with R. Fatoohi, of parallel algorithms for elliptic and parabolic partial differential equations has been completed. A paper reporting on the results of the implementation of parallel/concurrent algorithms for the Navier-Stokes equations on the MPP, FLEX/32, and CRAY/2 will be presented at the 1988 International Conference on Parallel Processing in addition to other papers which have appeared in journals.

Philip Hall

Work on the instability of compressible flows has been the main research topic carried out. With Peter Duck, the instability of the supersonic boundary layer on a axisymmetric body has been investigated. Duck and Hall (ICASE Report No. 88-10) discussed the

interaction of different axisymmetric TS instabilities of this flow. In a subsequent report, the interaction of non-axisymmetric modes is discussed (to appear as an ICASE report). With Gordon Erlebacher (High-Speed Aerodynamics Division, LaRC), the interaction for plane supersonic boundary layers has been discussed (to appear as an ICASE report). Work has continued on incompressible wave interaction problems. Hall and Mackerrell (ICASE Report No. 88-22) looked at the interaction of two and three dimensional TS waves near the leading edge of a swept wing. Hall and Mackerrell (ICASE Report No. 87-71) discussed the time dependent breakdown of large amplitude TS waves. Further work on the role of vortex structures in transition has continued with Frank Smith. The initial work, Hall and Smith (ICASE Report No. 87-25), has been extended to boundary layer flows and the later stages of channel flow transition (this work will appear in two ICASE reports). Work has begun on the instability of flows having shocks, with particular attention being focussed on the entropy layer instability problem for a hypersonic boundary layer.

M. Y. Hussaini

Investigations of instability and transition problems in canonical flows such as Poiseuille flow, Blasius flow and Taylor-Couette flow are continued. Final results of the numerical experiments in controlled incompressible boundary layers are available in ICASE Report No. 88-20. Study of Mach number effects on the secondary instabilities in boundary layers continued within the framework of both asymptotic theory and Navier-Stokes computations. Time-accurate numerical simulations of finite aspect-ratio Taylor-Couette flows indicate that the experimentally found hysteresis in the case of axisymmetric two-cell/four-cell exchange process is crucially dependent on the rate of spin-up or spin-down. A specific route to chaos in three-dimensional Taylor-Couette flow with fixed end-walls is being simulated. New subgrid-scale models, based on Favre-filtered fields, have been developed and are being tested against direct simulations. The preliminary results are to appear in an article in Physics of Fluids (ICASE Report No. 87-73).

This program of research is being carried out in collaboration with G. Erlebacher and T. A. Zang (High-Speed Aerodynamics Division, LaRC), and C. L. Streett (Transonic Aerodynamics Division, LaRC).

Kazufumi Ito

In a joint effort with R. Fabiano, the functional differential equation in a Hilbert space which arises in the modeling of linear viscoelastic beams of the Boltzman type is under study. We formulate the problem as an abstract Cauchy problem in the product space and several approximation schemes are developed. Well-posedness and convergence results are

given in the context of linear semigroup theory. This framework is especially useful for the study of parameter estimation and control problems.

In a joint effort with H. T. Banks, the feedback synthesis for boundary control problems is being investigated. The study involves the nontrivial extension of an approximation framework due to S. Gibson. Using the sesquilinear form/operator theoretic approach, we formulate a theoretical framework for solution of the linear quadratic regulator problems for a class of boundary control systems. This framework is used for construction and convergence analysis of approximation schemes. Further investigations will focus on construction of stabilizing finite dimensional compensators. The approach will use the approximation of Riccati solutions for both the linear quadratic regulator and the optimal observer.

T. L. Jackson and C. E. Grosch

An inviscid stability analysis is being performed to determine the growth rate of a disturbance as a function of Mach number, shear, temperature differences across the mixing layer, and heat release due to chemical mixing and heating. This is a critical problem in scramjet engine designs since it is necessary to understand the effect of chemistry on the transition process from laminar to turbulent flows in developing future scramjet engines.

David Kamowitz

An important consideration when using a numerical scheme to solve hyperbolic partial differential equations is the choice of boundary conditions. Even for such innocuous looking methods as Lax-Freidrichs a poor choice of boundary conditions can lead to trouble. Research has shown that for extrapolation schemes of degree higher than zero the scheme is no longer Total Variation Diminishing.

Another active area of research currently being conducted is joint with Naomi Decker. This work studies the use of multigrid techniques to increase the rate of convergence of hyperbolic conservation laws. To date the results are tentative, but encouraging.

Steve Keeling

Together with R. Silcox (Acoustics Division, LaRC) and H. T. Banks, the active suppression of cabin noise due to an advanced turbo-prop design is being considered within the framework of optimal control theory. First, the pressure field (p_n) due to the offending noise sources is assumed to have settled into a sinusoidal state. Further, p_n is assumed to be known throughout the cabin interior. Then the field (p_c) due to controlling sources (F) within the cabin is assumed to be governed by a nonhomogeneous wave equation in the interior and by a special boundary condition designed to conform with the experimentally observed frequency dependence in the reflection of monotone waves.

Since the initial boundary value problem formulated for p_c is not well-known, it has been written in a first order form for which the existence of a C° semigroup has been proved. Also, it is desirable for the generator to have a compact resolvent and for the semigroup to satisfy an exponential decay estimate. Both of these properties have recently been established.

The form of the controlling sources has been determined based upon the expectation that p_c should evolve into a sinusoidal state p_s . Specifically, p_s is assumed to be governed by the same equations as p_c , but with periodic boundary conditions. Then F is determined so that over a given period, $[p_s + p_n] + \lambda F$ is minimized. Finally, the result obtained from this minimization is used to govern p_c , even though the control is not optimal while $p_c \neq p_s$. However, it can be shown that $[p_c - p_s] \to 0$ as $t \to \infty$. Hence, this control strategy is stable, and asymptotically optimal.

Now, numerical approximations are being developed so that simulated results can be compared with those obtained experimentally. Plans also include a determination of the optimal placement of control sources. Further, the analysis will be refined with a state estimation procedure. Finally, this work will lead to an adaptive control mechanism.

3.15 2.15

Fumio Kojima

We continue to investigate the parameter identification technique for boundary value problems (BVP). The computational work in solving the identification of distributed parameter systems is often quite expensive. However, for a class of the boundary value problems, computational savings can be achieved in the use of boundary integral equation method (BIE). Since the BIE method makes it possible to replace BVP by certain integral equations, the related identification problems yield the effect of reducing the dimension of those problems. The purpose of this study is to find efficient algorithms for inverse problems arising in BVP by using the BIE method. For the identification of an integral equation of the second kind, we developed the numerical method with emphasis on the theoretical framework of the boundary parameter estimation technique. As spline collocation method was adopted for solving an integral equation in spline-based parameter space. We are currently pursuing studies for applying this method to the identification of boundary conditions and/or geometrical boundary shapes for 2-D elliptic and parabolic systems.

William D. Lakin

Integrating matrices provide the basis for a fast and efficient numerical technique for the solution of boundary value problems which arise in connection with studies of the stability of rotating flexible beams. Such beams comprise parts of many rotating structures whose stability must be insured. Previous applications of the integrating matrix technique have

been restricted to beams which are not subjected to point loadings. The present work, done jointly with R. Kvaternik (Loads and Aeroelasticity Division, LaRC), removes this restriction. In particular, the rotating beam may now have concentrated masses positioned along its length. Tip masses are also allowable.

A key feature of the generalized integrating matrix method is that including the concentrated masses does not increase the size of the final eigenvalue problem. Indeed, it is found that the matrices associated with the discrete masses can be derived separately and then simply added to the corresponding matrices for the continuous mass distribution to give the appropriate matrices for the full beam. Another attractive feature of the method is that discrete masses need not be located at grid points.

Accuracy of the extended technique has been assessed by considering critical rotation speeds for buckling in both the inplane and out-of-plane directions of a discrete mass representation for a rotating beam. In addition to validating the generalized integrating matrix method, the analysis for this problem has provided a new result for the behavior of the critical rotation speed for inplane buckling as the radius of rotation of the clamped end of the beam is reduced.

Nessan Mac Giolla Mhuiris

An attracting set for a dynamical system is a region in phase space which "attracts" nearby initial conditions. Any orbit started in the neighborhood of such a set, A, will evolve towards it and not leave A thereafter. Attracting sets for dissipative systems have dimensions which are less than those of the phase space as a whole and, as they eventually trap all initial conditions, it is their character which governs the long term asymptotic behavior of the system. Recently, examples have been found of some remarkable attractors. These strange attractors are characterized by the fact that orbits in them, which at some time lie infinitely close together, diverge from each other at an exponential rate and become uncorrelated in a finite time. In effect this precludes quantitative predictions of the behavior of individual orbits for all but the briefest times. Such behavior, termed sensitive dependence on initial conditions, must be present if the system is to be considered chaotic.

Many finite dimensional flows are now known to have this sensitive dependence on initial conditions, and it has been conjectured that the appearance of strange attractors is responsible for the evolution of turbulent fluids flows from their laminar precursors. A study is underway to address this conjecture. The rates at which nearby orbits diverge or converge for a given vector field is termed the Lyapunov spectrum of the flow. A strange attractor must have at least one positive and one negative Lyapunov exponent. A new method of measuring these exponents for the incompressible Navier-Stokes equations

has been formulated. This method is capable of measuring both positive and negative exponents. Currently, it is being applied to the case of spatially periodic flows in both two and three dimensions.

Nessan Mac Giolla Mhuiris and Yousuff Hussaini

There have been advances made in our understanding of the mechanisms of transition for flows in flat plate boundary layers, etc. As yet however there has been little progress made in the transition problem for flows in pipes of circular cross section. For this rather fundamental case it is believed that the important mechanisms are fully nonlinear and three dimensional in nature. This has meant that until recently numerical simulations could not hope to reproduce the necessary effects. We intend to utilize the Cray 2 supercomputer at the Ames Research Center to carry out a spectrally accurate simulation of these flows.

Dimitri Mavriplis

Work is continuing on the use of unstructured triangular meshes for solving steady-state compressible flow problems about complex configurations in two dimensions.

Research has focused on obtaining accurate and efficient solutions of the Euler equations for flows about arbitrary geometries in two dimensions. A study of the effects of artificial dissipation and boundary conditions on the accuracy of the inviscid solutions has been performed. An adaptive meshing technique has also been developed, enabling the accurate solution of complex flows with localized sharp gradients. The adaptive meshing technique has been combined with a full (unstructured) multigrid algorithm to yield an efficient Euler solver for unstructured meshes. Steady-state Euler solutions may be obtained in roughly 100 to 200 multigrid cycles, independently of the grid size. Results of this work, including inviscid transonic solutions about multi-element airfoils are to be presented at the First National Fluid Dynamics Conference (NFDC) in Cincinnati, July 1988.

Further increases in efficiency of the inviscid solver are being sought by attempting to reduce the amount of computational effort required within a multigrid cycle. To this end, a Fast Adaptive Composite (F.A.C.) multigrid algorithm has been developed for unstructured adaptive meshes. When a new fine mesh is generated, this method automatically determines the regions of the new mesh which have been refined or adapted as compared to the previous coarser mesh from which it was derived. In the multigrid cycle, time-stepping on the fine mesh(es) is performed only on these refined regions, thus omitting the task of time-stepping on regions of the fine mesh in which the resolution is unchanged from the previous coarser mesh.

An effort to develop a Reynolds-averaged Navier-Stokes solver on unstructured meshes is in the preliminary stages. To date, the effort has principally focused on the generation

of an unstructured (adaptive) triangular mesh suitable for high Reynolds number flows over airfoils. This requires the use of highly stretched mesh cells in the boundary layer and wake regions of the flow. The present method of generating a triangular grid, i.e., Delaunay triangulation, attempts to minimize cell aspect ratios, and thus does not easily allow for the construction of high aspect ratio triangles in these regions. The alternative being pursued consists of constructing the Delaunay triangulation in a mapped space or on a control surface in three-dimensional space, and then projecting the resulting triangulation back onto the physical space.

The implementation of unstructured mesh solvers on large scale (SIMD) parallel computers is also being investigated. This project is being performed in conjunction with Sherry Tomboulian. Preliminary work is aimed at implementing the basic unstructured mesh Euler solver on the Connection Machine, an SIMD machine (consisting of 64,000 processors) developed by Thinking Machines Corporation.

Piyush Mehrotra and John Van Rosendale

Our main research topic during the last six months has been the design of the parallel programming language, BLAZE 2. Though a strict super-set of the original BLAZE language, BLAZE 2 differs from it in many ways. In particular, BLAZE 2 is object-oriented, polymorphic, and allows streams and generators. The goal in BLAZE 2 was to create an effective language for highly parallel shared memory multiprocessors having the expressiveness of modern languages like CLU and Ada.

BLAZE 2 is being implemented by boot strapping, that is the BLAZE 2 compiler is being written in BLAZE 2. This approach was taken primarily as a way of studying the effectiveness of our language constructs. In order to bootstrap BLAZE 2, we have defined and implemented a compiler for a small subset of BLAZE 2, known as Flicker. We are currently in the process of implementing the BLAZE 2 parser in Flicker.

David Middleton

Several topics concerning fine-grained parallel computers are being studied. With Sherry Tomboulian, variations of the basic SIMD computer architecture are being investigated with regard to implementing various operations, especially communication. Research continues on the design of the FFP machine's communication system, with the intention of enabling useful computation in the message processors without noticeable impact on the time and hardware costs of message delivery and other system functions.

Other topics involve exploiting the virtual machine concept supported by the FFP machine. The FFP machine consists of a large number of simple processors, which may be divided, with few limitations, into disjoint groups that can support variably sized virtual

machines performing independent tasks. With Bruce Smith of the University of North Carolina, study of implementation issues for the OPS5 production system language continues. A compiler and simulator will be built to measure the actual speedup obtained by implementing the RETE matching algorithm with virtual machines. Also with Bruce Smith, a new study is beginning on using the associative processing facilities provided by virtual machines for manipulating sparse matrix systems.

Vijay K. Naik

Work is continuing towards developing methodologies that will characterize MIMD architectures and quantify the architecture and algorithm dependent parameters. We are concentrating mostly on problems related to scientific computation, although the principles developed here should be applicable to other appropriate areas of parallel processing as well. Various existing as well as proposed architectures and important algorithms are currently under investigation. Architectures include shared memory systems such as the Sequent and the Encore, message passing systems such as the Intel hypercube, and systems with both shared and local memories such as the BBN Butterfly, Flex/32, and the IBM RP3. The algorithms for which the data dependency analysis is carried out include direct methods such as Gaussian elimination based on the nested dissection ordering (jointly with Merrell Patrick), iterative schemes such as Jacobi, Gauss-Siedel, and Multigrid algorithms. Solutions of Poisson's equation, 2-D Navier-Stoke's equations, and the heat equation are used as the model problems. The ultimate goal is to develop techniques for automating the process of efficient parallel implementations.

In parallel to the above work, we are carrying on research to understand the effects of general data dependency patterns on the communication and synchronization overhead in architectures with shared and local memory. In the past few years extensive research has been done in developing schemes for minimizing communication and the synchronization costs and for improving the computational load balance for specific problems implemented on specific architectures. Our effort is towards generalizing these concepts.

David M. Nicol

Two model problems, a one-dimensional fluids simulation using dynamic regridding, and a battlefield simulation have been implemented on the FLEX/32 for the purpose of studying dynamic load balancing policies. These codes serve as testbeds for (1) centralized, static "risk averting" policies, (2) dynamic scheduling of risk averting policies, (3) dynamic remapping of exact balance policies, (4) decentralized balancing policies. This work is being done in collaboration with Joel Saltz and Jim Townsend (High-Speed Aerodynamics Division, LaRC).

Irregular and dynamic scientific problems are difficult to map onto parallel processors optimally. This research explores our ability to find polynomial-time mapping algorithms for constrained problem classes. This work is being done in collaboration with David O'Halloran (GE Research and Development Center), Rex Kincaid and Doug Shier (William and Mary), and Dana Richards (University of Virginia).

Research in reliable distributed systems has generally ignored the effect that workload mapping has on reliability. For example, software errors are more likely if two routines which share data are coresident than if they are separated—the additional risk occurs due to unintentional synchronization errors between those tasks. This project will examine the effects that mapping can have on reliability, and will develop mapping algorithms which take both performance, and reliability into account. This work is being done in collaboration with Phil Kearns and Steve Park (William and Mary).

Despite nearly a decade of study, high performance parallelized discrete-event simulations remain very difficult to construct, largely due to highly dynamic and irregular synchronization demands. We are investigating a synchronization technique called "appointments" which has proven to be effective on queueing network simulations operating under heavy loads. This work is being done in collaboration with Paul Reynolds (University of Virginia).

Merrell L. Patrick

Development of parallel algorithms for solving the generalized eigenvalue problem is continuing. Algorithms based on subspace iteration and spectrum slicing using Sturm sequences have been implemented and tested on the FLEX/32 at LaRC. Performance of these algorithms has been compared with that of a FLEX/32 implementation of the Lanczos method. Efforts to improve the performance of these algorithms are underway. One such effort requires the development of parallel methods for solving indefinite linear systems of equations and is underway in collaboration with Mark Jones.

In collaboration with R. Voigt and M. Jones, a comparative study of parallel programming environments available on the FLEX/32 system at NASA Langley was completed. The purpose of the study was to measure and compare the performance penalty associated with the different environments. The initial phase involved the implementation of Cholesky's method using ConCurrent FORTRAN supplied by Flexible Computing Corporation, the "force" developed by Harry Jordan, and PISCES developed by Terry Pratt. Both local and global memory versions of the method were programmed in each environment. Preliminary results indicate that FORCE and PISCES versions perform better than ConCurrent FORTRAN versions. An implementation using Schedule developed by D. Sorenson (Argonne National Laboratory) was added to the study. The incompleteness

of the FLEX/32 implementation of Schedule limited it's usefulness in this study. A study of the transportability of Force programs across different architectures is planned by running Force implementations of the Lanczos method on the FLEX/32 and CRAY 2. Work is continuing with Vijay Naik on the study of load distribution schemes which minimize the total data traffic required in Cholesky factorization of dense and sparse, symmetric, positive definite matrices on multiprocessor systems with local and shared memory. Efforts are being made to extend results obtained to a more general class of computations.

Terrence W. Pratt

The PISCES 2 parallel programming environment has been successfully installed on the NASA FLEX/32 parallel computer. The design of the system and directions for its use are described in two recent ICASE reports.

Performance measurements of the system are being made, in preparation for performance tuning and to better understand the effect on performance of the virtual machine provided by the PISCES 2 software. University of Virginia student Robert M. Wise has programmed in Pisces Fortran, 5 of the 16 problems proposed by Professor John Rice of Purdue in 1985. These problems form a useful test suite that has been programmed already for the FLEX and other parallel machines in various forms of "parallel FORTRAN."

A PISCES version of the NASA NICESPAR testbed is being implemented on the FLEX/32. We wish to understand the difficulties involved in moving large existing Fortran-based codes to parallel machines, and also we wish to understand the prospects for performance improvement in parallel versions of such large codes. The PISCES version of NICESPAR involves a minimal set of changes to the NICESPAR code, mainly at the top level-module and command-language. These changes implement the existing NICESPAR modules as PISCES tasks that communicate via messages. Synchronization is through requests for data base access, based on dependencies among the data sets requested from the data base by different modules.

Peter Protzel

Current research in software reliability and testing is continuing to investigate the performance characteristics of a new acceptance test method. The method is solely based on empirical data about the behavior of internal states of a program and does not require an a priori knowledge about the correct program behavior. Instead, a pseudo-oracle is used to gather statistical data about the frequency and values of the internal states under the normal (fault-free) program operation. This information can be used as an acceptance test for the application program by comparing the current values of internal states for each execution with the stored data.

In order to study the error detection performance, an experiment was performed by using the existing environment of a previous multi-version experiment that uses the launch interceptor problem as a model problem. Several versions instrumented with known single and multiple faults were executed together with an oracle to see if the occurred errors would have been detected by the acceptance test. Preliminary results show error detection rates between 82% and 100%, except for one case with a rate of 8.3%. Since the error detection rate was greater than zero in all cases, all faults were detected by the acceptance test. Further experiments with different faults are necessary to draw more general conclusions about the performance and applicability of the test. But the results obtained so far show promise for a certain class of applications, especially in the area of parallel programming, where a pseudo-oracle is often available in the form of a sequential version.

Other activities include the study of neural networks that can be used to solve hard optimization problems. Together with D. Palumbo (Information Systems Division, LaRC) we investigate the behavior of Hopfield-networks by solving the differential equations that describe the dynamics of the system. Since these networks usually produce only close-to-optimal solutions for a given problem, we have to analyze and understand the influence of the various parameter values on the system performance in order to develop more general guidelines of how to use neural networks effectively for different optimization problems.

I. Gary Rosen

We have developed an approximation framework for the solution of the discrete time linear quadratic regulator problem for abstract parabolic systems with unbounded input. Under relatively mild stabilizability, detectability, and approximation conditions, we were able to obtain a complete convergence theory for approximation to the optimal feedback gains. Our approach was based upon an operator theoretic formulation of the problem and the use of functional analytic techniques. We were able to successfully apply our abstract theory to a variety of boundary control systems including the Neumann boundary control of a heat/diffusion equation and the shear boundary control of a cantilevered viscoelastic beam.

In conjunction with Dr. D. S. Bernstein of the Harris Corporation we have been looking at the development of a finite dimensional approximation theory for the Bernstein/Hyland optimal projection approach to computing optimal fixed finite order compensators for distributed parameter control systems. The technique has been tested and shown to perform well on one dimensional single input/single output parabolic and hereditary systems. Both our theoretical and our numerical studies related to this effort are continuing.

Joel H. Saltz and David Nicol

Many problems are characterized by a high degree of potential data level parallelism. In many cases, workloads cannot be fully characterized during compilation due to data dependencies that become manifest only at runtime. In these cases, we must utilize methods applied during the program's execution to parallelize the work specified by the program. We are designing and implementing an automated runtime system, designed to be interfaced to high level languages. The system seeks to efficiently parallelize, cluster, or aggregate and math workloads from scientific computations.

Three basic runtime aggregation strategies are currently being pursued. When the user desires, we allow the specification of an explicit high level strategy. We are also developing a variety of runtime aggregation strategies. Some of these make implicit use of the substantial degree of regularity often present in many problems. Others do not depend on the existence of any particular pattern of data dependencies. All of these strategies are generally parameterized so that the granularity can be specified in a straightforward, machine independent manner.

Experimental work to characterize aggregation strategies has been performed on the Encore Multimax, the FLEX/32, and the Intel iPSC. The problems examined were quite realistic. We utilized a time driven discrete event simulation, a fluid dynamics simulation code employing an adaptive mesh, and codes that parallelized the solution to very sparse triangular systems of linear equations. Methods of obtaining parameterized tradeoffs between load balance and the costs of synchronization and/or communication were proposed and tested, and methods of scheduling problem mappings were investigated. Much of this work has been recently reported in ICASE Reports 87-22, 87-39, and 87-52. We are now in the process of generalizing these aggregation and scheduling methods.

We are also in the process of defining and constructing a C based intermediate language to be used between the compilers and the runtime system. This intermediate language is also being designed to be usable in a stand-alone manner.

A directed acyclic graph describing the parallelism available is generated when the program is executed, and the contents of the partition tuples are decided using generalized versions of the aggregation strategies described above.

Paul Saylor

Stability studies of a solution procedure for the flow equations reduce to a generalized eigenvalue problem for a large banded sparse matrix. Only the extreme eigenvalues of the matrix need be computed. The conventional power method is one algorithm; another algorithm is the Arnoldi method. These algorithms (or the mathematical equivalent) are part of an iterative solver package, Chebycode, developed at the University of Illinois,

Los Alamos National Laboratory, and Sandia National Laboratory by Steve Ashby. The purpose of Chebycode, however, is the solution of linear sets rather than computing eigenvalues. As a result, the documentation does not describe the appropriate parameter choice for computing extreme eigenvalues. Two memos were written to supplement the documentation.

Applications of iterative methods to a matrix arising in the modeling of a moving grid were discussed. If the matrix were inverted, the inverse would allow easy computations for a variety of conditions. However, if the matrix originates from a 3D problem, it is too large to invert.

An implicit formulation of the Transonic Small Disturbance Equation was outlined. An implicit formulation requires the solution of a large nonsymmetric linear set.

Numerical tests with polynomial preconditioning were described. Polynomial preconditioning of the conjugate gradient appears attractive on vector and parallel machines. The experiments, also made by Steve Ashby at the University of Illinois and Los Alamos, suggest a speedup on the Cray-XMP for large, sparse, badly conditioned positive definite matrices.

Charles Speziale

Work was conducted on the correlation of results from direct numerical simulations of compressible isotropic turbulence with eddy shocklets in collaboration with M. Y. Hussaini, T. Zang (High-Speed Aerodynamics Branch, LaRC), and G. Erlebacher (High-Speed Aerodynamics Branch, LaRC). The testing of subgrid scale models developed by the same authors for the large-eddy simulation of compressible turbulent flows with eddy shocklets is currently underway. The large density gradients present in such supersonic flows provides a stringent test for any turbulence model. An improved nonlinear K-ε model has been finalized for applications to rotating and curved turbulent flows (ONR Contract N00014-85-K-0238). This model was shown to yield substantially improved predictions for homogeneous turbulent shear flow in a rotating framework. Improved second-order closure models for homogeneous turbulence are currently being developed in collaboration with Nessan Mac Giolla Mhuiris based on a dynamical systems approach. In a previous study (ICASE Report No. 85-49), it was shown that the commonly used turbulent second-order closure models can give rise to spurious predictions in rapidly rotating flows.

Charles Speziale and Nessan Mac Giolla Mhuiris

We are comparing the behavior of some commonly used turbulence models for the test problem of homogeneous turbulent shear flow in a rotating frame. For such flows the fundamental physical parameter is Ω/s the ratio of the rotation rate to the shear rate.

In this study, particular emphasis is placed on the bifurcations of the equilibrium states which occur as this parameter is varied.

A major defect of the simple K- ε model is that it is completely unaffected by changes in the bifurcation parameter. On the other hand, both of the second order closures investigated (the Launder, Reece and Rodi model, and the Rotta-Kolmogorov model) have a built in dependence on Ω/s . In fact, the bifurcation diagrams obtained for these models are qualitatively identical. Interestingly, these diagrams are not inconsistent with the data available in the literature from experimentation and computer simulation.

On the other hand, the quantitative predictions of the models are flawed. In the next stage of our study, we plan to improve the models in this regard.

Shlomo Ta'asan

Research is being conducted on developing multigrid methods for bifurcation problems and identification problems. In the first subject, the focus is on bifurcation for non-symmetric problems. Recently, research on the symmetric case has been completed (see ICASE Report No. 87-40). Extension of the techniques presented there for the non-symmetric case is under development. Moreover, some new questions like the efficient computation of Hopf bifurcation points and fast calculation of branches of periodic solutions are under study. When these techniques are developed a possible application on which they may be demonstrated is a "vortex breakdown" simulation (which was the original motivation for this research).

In the other direction of research the focus is on problems arising in thermal non-destructive evaluation. Here one is interested in estimating flaws in materials from observation of temperature maps of the object to be considered. Modelling has been developed in the first stage of the research. The efficient numerical solution of the minimization problems that were constructed in the modelling stage is under study. The techniques will be demonstrated with some real data related to the shuttle boosters.

Eitan Tadmor

In recent years spectral methods have become one of the standard tools for the approximate solution of nonlinear conservation laws. It is well-known that the spectral methods enjoy high order of accuracy whenever the underlying solution is smooth. On the other hand, one of the main disadvantages in using spectral methods for nonlinear conservation laws, lies in the formation of Gibbs phenomena, once spontaneous shock discontinuities appear in the solution; the global natrue of spectral methods then pollutes the unstable Gibbs oscillations over all the computational domain which prevents the convergence of spectral approximation in these cases. One of the standard techniques to mask the oscilla-

tory behavior of spectral approximations is based on spectrally accurate post-processing of these approximations. Indeed, the convergence of such recovery techniques was justified by linear arguments in our previous work [ICASE Report No. 85-38]. However, for nonlinear problems we show by a series of prototype counterexamples, that spectral solutions with or without such post-processing techniques, do not converge to the 'physically' correct entropy solutions of the conservation laws. The main reason for this failure of convergence of spectral methods is explained by their lack of entropy dissipation.

A similar situation which involves unstable oscillations, is encountered with finite-difference approximations to nonlinear conservation laws. Here, the problem of oscillations is usually solved by the so-called vanishing viscosity method. Namely, one adds artificial viscosity, such that on the one hand it retains the formal accuracy of the basic scheme, while on the other hand, it is sufficient to stabilize the Gibbs oscillations. The Spectral Viscosity Method (SVM) proposed in [ICASE Report No. 87-54] see also [ICASE Report No. 88-4] attempts, in an analogous way, to stabilize the Gibbs oscillations and consequently to guarantee the convergence of spectral approximations, by augmenting them with proper viscous modifications.

Currently, we develop a complete convergence theory for general discrete approximations (finite-difference, finite-element and spectral methods) for nonlinear system of conservation law, a convergence theory which grew from the analysis of the above mentioned SVM. We introduce the notion of consistency of such approximations (with both the PDE and the augmenting entropy condition) and show that consistency together with L^{∞} -stability imply convergence for scalar as well as a wide class of 2×2 systems. The modern finite-difference shock capturing techniques (high-resolution TVD methods, for example), the finite-element streamline-diffusion methods and the Spectral Viscosity Method, are shown to be consistent and (in the scalar case) L^{∞} -stable, and hence their convergence follows.

Hillel Tal-Ezer

In applying an iterative algorithm to solve a linear system Ax = b, we frequently encounter the situation where the algorithm is slow to converge. A standard approach is to use the idea of preconditioning. It is done by considering an equivalent system Cx = d such that by applying the same algorithm we will increase the rate of convergence. In the present research we show that in general there is no correspondence between the condition number of the matrix and the rate of convergence of the iterative algorithm. A particular example of this observation is related to the normal equations A'Ax = A'b. In this case C = A'A and even though the condition number is squared (when A is normal), the rate of convergence can be reduced significantly.

Polynomial interpolation is an essential subject in numerical analysis. Dealing with a real interval, it is well-known that even if f(x) is an analytic function, interpolating at equally spaced points can diverge .On the otherhand, interpolating at the zeroes of the corresponding Chebyshev polynomial will converge. Using the Newton formula, this result of convergence is true only on the theoretical level. In this research we show that the algorithm which computes the divided differences is numerically stable only if: 1.) The interpolating points are arranged in a certain order. 2.) The size of the interval is 4.

This phenomenon of numerical instability of the Newton formula is treated in the general context of interpolation in the complex plane. Based on the two modifications mentioned above we describe several stable algorithms which enable us to construct interpolating polynomials of very large degree. The need for such polynomials arises in approximating a finite operator which can be presented as f(A) where A is another finite operator. (The particular case f(z) = exp(z) is widely treated in the numerical analysis literature.) Approximating f(A) is reduced to a problem of approximating f(z) where z belongs to a domain D in the complex plane which includes all the eigenvalues of A and the approximation problem is approached via interpolation.

Sherry Tomboulian

A spectral method code for solving incompressible Navier-Stokes equations has been implemented on the Connection Machine 2 in collaboration with Craig Streett (Transonic Aerodynamics Division, LaRC) and Michele Macaraeg (High-Speed Aerodynamics Division, LaRC). Preliminary timing results indicate that the method is competitive with a Cray 2.

In collaboration with Dave Middleton and Tom Crockett, alternative programming environments for programming the Navier-Stokes computer (a reconfigurable pipelined architecture) are being explored. A paper on this topic was accepted for the 1988 International Conference on Parallel Processing.

Work is continuing on methods for graph embedding and communication in SIMD architecture. Empirical data is being collected for an extension to the basic method which allows a branching path structure from a single vertex similar to dendritic growth. The method is being examined as a method for implementing neural networks. Designs are underway for software implementation that will run on either the Connection Machine or the Blitzen architecture. The design aims for a three layer neural network with 16,000 neurons in each level, each neuron having about 100 connections.

In collaboration with Dimitri Mavriplis, ways of implementing unstructured mesh multigrid problems on massively parallel architectures are under investigation. We have considered using the above mentioned method of graph embedding to implement the problem on the Blitzen machine, and conducted some simulations to determine the suitability of the graph embedding method. Indications are positive, but since the Blitzen hardware is not available, we have decided to proceed with implementation on the CM2, using the hardware router instead of the graph embedding techniques implemented in software.

Methods of extending SIMD architectures are being explored with Dave Middleton and John Van Rosendale. We are particularly interested in the blending of SIMD and MIMD approaches.

Eli Turkel

Work is continuing on the multi-dimensional Runge-Kutta multigrid algorithm to compute steady state inviscid and viscous flows. As before much of the work is concentrating on the artificial viscosity. Increasing the artificial viscosity accelerates the convergence to the steady state but at the expense of reduced accuracy in the final steady state solution. An AIAA paper was finished together with D. Caughey of Cornell University discussing the advantages and disadvantages of applying the artificial viscosity to the total energy or total enthalpy. Further improvements were made in the three dimensional case and the Euler calculations have been carried out for a range of wing configurations and a range of Mach numbers, subsonic, transonic, and supersonic. In addition results have also been obtained for some viscous flows using the multigrid code. A report is presently being written together with V. Vatsa (Transonic Aerodynamics Division, LaRC) and B. Wedan (Vigyan Research Association, Inc.). Work has also begun on the use of a matrix artificial viscosity that will bring the code closer to the upwind codes. Further analysis is being carried out comparing central difference schemes with an artificial viscosity and upwind TVD schemes.

A new project has begun in analyzing the use of chaotic schemes to solve hyperbolic and parabolic equations to a steady state. In parallel computers one of the difficulties is the communication between different processors. In chaotic or asynchronous schemes this difficulty is eliminated by updating each node without waiting for the neighbors to be updated. This requires that the new scheme have convergence rates which are not significantly worse than those of standard synchronous schemes. Some preliminary analysis has lead to the investigation of stochastic partial differential equations. In addition computational tests are being preformed for a variety of schemes for both hyperbolic and parabolic equations. These include the standard schemes plus Runge-Kutta type methods and also multigrid methods. Applications to fluid dynamics will be studied in the future.

Another project which is being started together with A. Bayliss is the effect of transformations of grids on the approximation of functions with applications to spectral methods. In particular we wish to extend the work of A. Bayliss on adaptive Chebyshev schemes.

As such we are analyzing different transformations and also different ways of measuring the accuracy of a scheme.

REPORTS AND ABSTRACTS

LeVeque, Randall J.: High resolution finite volume methods on arbitrary grids via wave propagation. ICASE Report No. 87-68, October 20, 1987, 29 pages. Submitted to J. Comput. Phys.

In previous work by the author, a generalization of Godunov's method for systems of conservation laws has been developed and analyzed that can be applied with arbitrary time steps on arbitrary grids in one space dimension. Stability for arbitrary time steps is achieved by allowing waves to propagate through more than one mesh cell in a time step. In this paper the method is extended to second order accuracy and to a finite volume method in two space dimensions. This latter method is based on solving one dimensional normal and tangential Riemann problems at cell interfaces and again propagating waves through one or more mesh cells. By avoiding the usual time step restriction of explicit methods, it is possible to use reasonable time steps on irregular grids where the minimum cell area is much smaller than the average cell.

Boundary conditions for the Euler equations are discussed and special attention is given to the case of a Cartesian grid cut by an irregular boundary. In this case small grid cells arise only near the boundary, and it is desirable to use a time step appropriate for the regular interior cells. Numerical results in two dimensions show that this can be achieved.

Speziale, Charles G.: On helicity fluctuations and the energy cascade in turbulence. ICASE Report No. 87-69, November 4, 1987, 11 pages. Submitted to Physical Review Letters.

Recent conjectures concerning the correlation between regions of high local helicity and low dissipation are examined from a rigorous theoretical standpoint based on the Navier-Stokes equations. It is proven that only the solenoidal part of the Lamb vector $\omega \times u$ (which is directly tied to the nonlocal convection and stretching of vortex lines) contributes to the energy cascade in turbulence. Consequently, it is shown that regions of low dissipation can be associated with either low or high helicity—a result which disproves earlier speculations concerning either low or high helicity—a result which disproves earlier speculations concerning this direct connection between helicity and the energy cascade. Some brief examples are given along with a discussion of the consistency of these results with the most recent computations of helicity fluctuations in incompressible turbulent flows.

Bernardi, Christine, Naima Debit, and Yvon Maday: Coupling finite element and spectral methods: First results. ICASE Report No. 87-70, November 4, 1987, 31 pages. Submitted to Math. Comp.

A Poisson equation on a rectangular domain is solved by coupling two methods: the domain is divided in two squares, a finite element approximation is sued on the first square and a spectral discretization is used on the second one. Two kinds of matching conditions on the interface are presented and compared: in both cases, error estimates are proved.

Hall, Philip and Sharon MacKerrell: On the onset of three-dimensionality and time-dependence in the Görtler vortex problem. ICASE Report No. 87-71, November 4, 1987, 38 pages. Submitted to J. Fluid Mech.

The instability of large amplitude Görtler vortices in a growing boundary layer is discussed in the fully nonlinear regime. It is shown that a three-dimensional breakdown to a flow with wavy vortex boundaries similar to that which occurs in the Taylor vortex problem takes place. However, the instability is confined to the thin shear layers which were shown by Hall and Lakin (1987) to trap the region of vortex activity. The disturbance eigenfunctions decay exponentially away from the center of these layers so that the upper and lower shear layers can support independent modes of instability. The structure of the instability, in particular its location and speed of downstream propagation, is found to be entirely consistent with recent experimental results. Furthermore, it is shown that the upper and lower layers support wavy vortex instabilities with quite different frequencies. This result is again consistent with the available experimental observations.

Sunahara, Y. and F. Kojima: A method of boundary parameter estimation for a two-dimensional diffusion system under noisy observations. ICASE Report No. 87-72, November 10, 1987, 27 pages. Submitted to the Proc. of the 10th IFAC World Congress on Automatic Control, Munich, FRG 1987.

The purpose of this paper is to establish a method for identifying unknown parameters involved in the boundary state of a class of diffusion systems under noisy observations. A mathematical model of the system dynamics is given by a two-dimensional diffusion equation, whose boundary condition is partly unknown due to the existence of an unknown parameter. Noisy observations are made by sensors allocated on the system boundary. Starting with the mathematical model mentioned above, an on-line parameter estimation algorithm is proposed within the framework of the maximum likelihood estimation. Existence of the optimal solution and related necessary conditions are discussed. By solving a local variation of the cost functional with respect to the perturbation of parameters, the estimation mechanism is proposed in a form of recursive computations. Finally, the feasibility of the estimator proposed here is demonstrated through results of digital simulation experiments.

Speziale, C. G., G. Erlebacher, T. A. Zang, and M. Y. Hussaini: On the subgrid-scale modeling of compressible turbulence. ICASE Report No. 87-73, December 7, 1987, 13 pages. Submitted to Physics of Fluid.

A subgrid-scale model recently derived by Yoshizawa for use in the large-eddy simulation of compressible turbulent flows is examined from a fundamental theoretical and computational standpoint. It is demonstrated that this model, which is only applicable to compressible turbulent flows in the limit of small density fluctuations, correlates somewhat poorly with the results of direct numerical simulations of compressible isotropic turbulence at low Mach numbers. An alternative model, based on Favre-filtered fields, is suggested which appears to reduce these limitations.

Speziale, Charles G. and Tuan Ngo: Numerical solution of turbulent flow past a backward facing step using a nonlinear K-ε model. ICASE Report No. 87-74, December 7, 1987, 45 pages. Submitted to ASME J. Applied Mech.

The problem of turbulent flow past a backward facing step is important in many technological applications and has been used as a standard test case to evaluate the performance of turbulence models in the prediction of separated flows. It is well known that the commonly used $K-\varepsilon$ (and $K-\ell$) models of turbulence yield inaccurate predictions for the reattachment point in this problem. By an analysis of the mean vorticity transport equation, it will be

argued that the intrinsically inaccurate prediction of normal Reynolds stress differences by the K- ε and K- ℓ models is a major contributor to this problem. Computations using a new nonlinear K- ε model (which alleviates this deficiency) are made with the TEACH program. Comparisons are made between the improved results predicted by this nonlinear K- ε model and those obtained from the linear K- ε model as well as from second-order closure models.

Alaghband, Gita: Parallel pivoting combined with parallel reduction. ICASE Report No. 87-75, December 8, 1987, 40 pages. Submitted to Parallel Computing Journal.

Parallel algorithms for triangularization of large, sparse, and unsymmetric matrices are presented. The method combines the parallel reduction with a new parallel pivoting technique, control over generations of fill-ins and check for numerical stability, all done in parallel with the work being distributed over the active processes. The parallel technique uses the compatibility relation between pivots to identify parallel pivot candidates and uses the compatibility relation between pivots to identify parallel pivot candidates and uses the Markowitz number of pivots to minimize fill-in. This technique is not a preordering of the sparse matrix and is applied dynamically as the decomposition proceeds.

Brewer, D. W. and J. S. Gibson: Parameter identification and sensitivity analysis for a robotic manipulator arm. ICASE Report No. 88-1, December 22, 1987, 30 pages. Submitted to Internat. J. Robotics Research.

This paper describes the development of a nonlinear dynamic model for large oscillations of a robotic manipulator arm about a single joint. Optimization routines are formulated and implemented for the identification of electrical and physical parameters from dynamic data taken from an industrial robot arm. Special attention is given to difficulties caused by large sensitivity of the model with respect to unknown parameters. Performance of the parameter identification algorithm is improved by choosing a control input that allows actuator emf to be included in an electro-mechanical model of the manipulator system.

Nicol, David M.: Parallel algorithms for mapping pipelined and parallel computations. ICASE Report No. 88-2, April 1, 1988, 34 pages. Submitted to IEEE Trans. Comput.

Many computational problems in image processing, signal processing, and scientific computing are naturally structured for either pipelined or parallel computation. When mapping such problems onto a parallel architecture, it is often necessary to aggregate an ovbious problem decomposition. Even in this context the general mapping problem is known to be computationally intractable, but recent advances have been made in identifying classes of problems and architectures for which optimal solutions can be found in polynomial time. Among these, the mapping of pipelined or parallel computations onto linear array, shared memory, and host-sattelite systems figures prominently. This paper extends that work first by showing how to improve existing serial mapping algorithms. Our improvements have significantly lower time and space complexities: in one case we reduce a published $O(nm^3)$ time algorithm for mapping m modules onto n processors to an $O(nm\log m)$ time complexity, and reduce its space requirements from $O(nm^2)$ to O(m). We then reduce run-time complexity further with parallel mapping algorithms based on these improvements that run on the architectures for which they are creating mappings.

Tomboulian, Sherryl: Introduction to a system for implementing neural net connections on SIMD architectures. ICASE Report No. 88-3, January 11, 1988, 15 pages. Submitted to Proc. 1987 Conference on Neural Information Processing. Natural and Synthetic, published by A.I.P.

Neural networks have attracted much interest recently, and using parallel architectures to simulate neural networks is a natural and necessary application. The SIMD model of parallel computation is chosen because systems of this type can be built with large numbers of processing elements. However, such systems are not naturally suited to generalized communication. A method is proposed that allows an implementation of neural network connections on massively parallel SIMD architectures. The key to this system is an algorithm that allows the formation of arbitrary connections between the "neurons." A feature is the ability to add new connections quickly. It also has error re covery ability and is robust over a variety of network topologies. Simulations of the general connection system, and its implementation on the Connection Machine, indicate that the time and space requirements are proportional to the product of the average number of connections per neuron and the diameter of the interconnection network.

Maday, Yvon and Eitan Tadmor: Analysis of the spectral vanishing viscosity method for periodic conservation laws. ICASE Report No. 88-4, January 11, 1988, 36 pages. Submitted to SIAM J. Numer. Anal.

We analyze the convergence of the spectral vanishing method for both the spectral and pseudospectral discretizations of the inviscid Burgers' equation. We prove that this kind of vanishing viscosity is responsible for a spectral decay of those Fourier coefficients located toward the end of the computed spectrum; consequently, the discretization error is shown to be spectrally small independent of whether the underlying solution is smooth or not. This in turn implies that the numerical solution remains uniformly bounded and convergence follows by compensated compactness arguments.

Fatoohi, Raad A. and Chester E. Grosch: Implementation and analysis of a Navier-Stokes algorithm on parallel computers. ICASE Report No. 88-5, January 11, 1988, 31 pages. Submitted to 1988 International Conference on Parallel Processing.

This paper presents the results of the implementation of a Navier-Stokes algorithm on three parallel/vector computers. The object of this research is to determine how well, or poorly, a single numerical algorithm would map onto three different architectures. The algorithm is a compact difference scheme for the solution of the incompressible, two-dimensional, time dependent Navier-Stokes equations. The computers were chosen so as to encompass a variety of architectures. They are: the MPP, an SIMD machine with 16K bit serial processors; Flex/32, an MIMD machine with 20 processors; and Cray/2. The implementation of the algorithm is discussed in relation to these architectures and measures of the performance on each machine are given. The basic comparison is among SIMD instruction parallelism on the MPP, MIMD process parallelism on the Flex/32, and vectorization of a serial code on the Cray/2. Simple performance models are used to describe the performance. These models highlight the bottlenecks and limiting factors for this algorithm on these architectures. Finally conclusions are presented.

Tomboulian, Sherryl, Thomas W. Crockett, and David Middleton: A visual programming environment for the Navier-Stokes computer. ICASE Report No. 88-6, January 11, 1988, 18 pages. Submitted to 1988 International Conference on Parallel Processing.

The Navier-Stokes computer is a high-performance, reconfigurable, pipelined machine designed to solve large computational fluid dynamics problems. Due to the complexity of the architecture, development of effective high-level language compilers for the system appears to be a very difficult task. Consequently, a visual programming methodology has been developed which allows users to program the system at an architectural level by constructing diagrams of the pipeline configuration. These schematic program representations can then be checked for validity and automatically translated into machine code. The visual environment is illustrated by using a prototype graphical editor to program an example problem.

Saylor, Paul E.: Leapfrog variants of iterative methods for linear algebraic equations. ICASE Report No. 88-7, January 13, 1988, 63 pages. Submitted to SIAM J. Sci. Stat. Comput.

Two iterative methods are considered, Richardson's method and a general second order method. For both methods, a variant of the method is derived for which only even numbered iterates are computed. The variant is called a leapfrog method. Comparisons between the conventional form of the methods and the leapfrog form are made under the assumption that the number of unknowns is large. In the case of Richardson's method, it is possible to express the final iterate in terms of only the initial approximation, a variant of the iteration called the grand-leap method. In the case of the grand-leap variant, a set of parameters is required. An algorithm is presented to compute these parameters that is related to algorithms to compute the weights and abscissas for Gaussian quadrature. General algorithms to implement the leapfrog and grand-leap methods are presented. Algorithms for the important special case of the Chebyshev method are also given.

Henshaw, W. D., H. O. Kreiss, and L. G. Reyna: On the smallest scale for the incompressible Navier-Stokes equations. ICASE Report No. 88-8, January 20, 1988, 48 pages. Submitted to Theoretical and Computational Fluid Dynamics.

We prove that for solutions to the two and three dimensional incompressible Navier-Stokes equations the minimum scale is inversely proportional to the square root of the Reynolds number based on the kinematic viscosity and the maximum of the velocity gradients. The bounds on the velocity gradients can be obtained for two dimensional flows, but have to be assumed in three dimensions. Numerical results in two dimensions are given which illustrate and substantiate the features of the proof. Implications of the minimum scale result to the decay rate of the energy spectrum are discussed.

Fennell, Robert E.: An application of eigenspace methods to symmetric flutter suppression. ICASE Report No. 88-9, January 21, 1988, 21 pages. Submitted to Proc. AIAA Guidance, Navigation, and Control Conference.

An eigenspace assignment approach to the design of parameter insensitive control laws for linear multivariable systems is presented. The control design scheme utilizes flexibility in eigenvector assignments to reduce control system sensitivity to changes in system parameters. The methods involve use of the singular value decomposition to provide an exact description of allowable eigenvectors in terms of a minimum number of design parameters. In a design example, the methods are applied to the problem of symmetric flutter suppression in an aeroelastic vehicle. In this example the flutter mode is sensitive to changes in dynamic pressure and eigenspace methods are used to enhance the performance of a stabilizing minimum energy/linear quadratic regulator controller and associated observer.

Results indicate that the methods provide feedback control laws that make stability of the nominal closed loop systems insensitive to changes in dynamic pressure.

Duck, P. W. and P. Hall: On the interaction of Tollmien-Schlichting waves in axisymmetric supersonic flows. ICASE Report No. 88-10, January 21, 1988, 20 pages. Submitted to Quarterly Journal of Mechanics and Applied Math.

It is known that two-dimensional lower branch Tollmien-Schlichting waves described by triple-deck theory are always stable for planar supersonic flows. Here the possible occurrence of axisymmetric unstable modes in the supersonic flow around an axisymmetric body is investigated. In particular flows around bodies with typical radii comparable with the thickness of the upper deck are considered. It is shown that such unstable modes exist below a critical nondimensional radius of the body a_0 . At values of the radius above a_0 all the modes are stable whilst if unstable modes exist they are found to occur in pairs. The interaction of these modes in the nonlinear regime is investigated using a weakly nonlinear approach and it is found that, dependent on the frequencies of the imposed Tollmien-Schlichting waves, either of the modes can be set up.

Quarteroni, Alfio and Giovanni Sacchi-Landriani: Domain decomposition preconditioners for the spectral collocation method. ICASE Report No. 88-11, January 22, 1988, 37 pages. Submitted to J. Scientific Computing.

We propose and analyze several block iteration preconditioners for the solution of elliptic problems by spectral collocation methods in a region partitioned into several rectangles. It is shown that convergence is achieved with a rate which does not depend on the polynomial degree of the spectral solution. The iterative methods here presented can be effectively implemented on multiprocessor systems due to their high parallelism degree.

Banks, H. T. and K. Ito: A unified framework for approximation in inverse problems for distributed parameter systems. ICASE Report No. 88-12, January 25, 1988, 33 pages. Submitted to Control Theory and Advanced Technology.

We present a theoretical framework that can be used to treat approximation techniques for very general classes of parameter estimation problems involving distributed systems that are either first or second order in time. Using the approach developed, one can obtain both convergence and stability (continuous dependence of parameter estimates with respect to the observations) under very weak regularity and compactness assumptions on the set of admissible parameters. This unified theory can be used for many problems found in the recent literature and in many cases offers significant improvements to existing results.

Abarbanel, Saul and Ajay Kumar: Compact high order schemes for the Euler equations. ICASE Report No. 88-13, February 10, 1988, 16 pages. Submitted to J. of Scientific Computing, D. Gottlieb (ed.).

An implicit approximate factorization (AF) algorithm is constructed which has the following characteristics.

• In 2-D: The scheme is unconditionally stable, has a 3 × 3 stencil and at steady state has a fourth order spatial accuracy. The temporal evolution is time accurate either to 1st or 2nd order through choice of parameter.

• In 3-D: The scheme has almost the same properties as in 2-D except that it is now only conditionally stable, with the stability condition (the CFL number) being dependent on the "cell aspect ratios," Wy/Wx and Wz/Wx.

The stencil is still compact and fourth order accuracy at steady state is maintained.

Numerical experiments on a 2-D shock-reflection problem show the expected improvement over lower order schemes, not only in accuracy (measured by the L₂ error) but also in the dispersion. It is also shown how the same technique is immediately extendable to Runge-Kutta type schemes resulting in improved stability in addition to the enhanced accuracy.

Naik, Vijay K. and Merrell L. Patrick: Data traffic reduction schemes for sparse Cholesky factorizations. ICASE Report No. 88-14, February 10, 1988, 34 pages. Submitted to Int. Supercomputing Conf.

Load distribution schemes are presented which minimize the total data traffic in the Cholesky factorization of dense and sparse, symmetric, positive definite matrices on multiprocessor systems with local and shared memory. The total data traffic in factoring an n \times n sparse, symmetric, positive definite matrix representing an n-vertex regular 2-D grid graph using n_a^a , $a \leq 1$, processors is shown to be $0(n^{1+\frac{a}{2}})$. It is $0(n^{\frac{a}{2}})$, when n^a , $a \geq 1$, processors are used. Under the conditions of uniform load distribution these results are shown to be asymptotically optimal. The schemes allow efficient use of up to 0(n) processors before the total data traffic reaches the maximum value of $0(n^{\frac{a}{2}})$. The partitioning employed within the scheme, allows a better utilization of the data accessed from shared memory than those of previously published methods.

Gottlieb, David and Richard S. Hirsh: Parallel pseudospectral domain decompositon techniques. ICASE Report No. 88-15, February 17, 1988, 28 pages. Submitted to SISC.

The influence of interface boundary conditions on the ability to parallelize pseudospectral multidomain algorithms is investigated. Using the properties of spectral expansions, a novel parallel two domain procedure is generalized to an arbitrary number of domains each of which can be solved on a separate processor. This interface boundary condition considerably simplifies influence matrix techniques.

Koelbel, Charles, Piyush Mehrotra, John Van Rosendale: Semi-automatic process partitioning for parallel computation. ICASE Report No. 88-16, February 22, 1988, 22 pages. Submitted to Parallel Programming.

On current multiprocessor architectures one must carefully distribute data in memory in order to achieve high performance. Process partitioning is the operation of rewriting an algorithm as a collection of tasks, each operating primarily on its own portion of the data, to carry out the computation in parallel. In this paper we consider a semi-automatic approach to process partitioning in which the compiler, guided by advice from the user, automatically transforms programs into such an interacting task system. This approach is illustrated with a picture processing example written in BLAZE, which is transformed into a task system maximizing locality of memory reference.

Yang, Y. P. and J. S. Gibson: Adaptive control of a manipulator with a flexible link. ICASE Report No. 88-17, February 26, 1988, 19 pages. Submitted to Journal of Robotic Systems.

An adaptive controller for a manipulator with one rigid link and one flexible link is presented. The performance and robustness of the controller are demonstrated by numerical simulation results. In the simulations, the manipulator moves in a gravitational field and a finite element model represents the flexible link.

Speziale, Charles G.: Turbulence modeling in non-inertial frames of reference. <u>ICASE</u> Report No. 88-18, March 3, 1988, 51 pages. Submitted to J. Fluid Mech.

The effect of an arbitrary change of frame on the structure of turbulence models is examined from a fundamental theoretical standpoint. It is proven, as a rigorous consequence of the Navier-Stokes equations, that turbulence models must be form invariant under arbitrary translational acclerations of the reference frame and should only be affected by rotations through the intrinsic mean vorticity. A direct application of this invariance property along with the Taylor-Proudman Theorem, material frame-indifference in the limit of two-dimensional turbulence and Rapid Distortion Theory is shown to yield powerful constraints on the allowable form of turbulence models. Most of the commonly used turbulence models are demonstrated to be in serious violation of these constraints and consequently are inconsistent with the Navier-Stokes equations in non-inertial frames. Alternative models with improved non-inertial properties are developed and some simple applications to rotating turbulent flows are considered.

Gill, Doron and Eitan Tadmor: An $O(N^2)$ method for computing the eigensystem of $N \times N$ symmetric tridiagonal matrices by the divide and conquer approach. ICASE Report No. 88-19, March 4, 1988, 24 pages. Submitted to SIAM J. Sci. Statist. Comput.

We propose an efficient method to solve the eigenproblem of $N \times N$ Symmetric Tridiagonal (ST) matrices. Unlike the standard eigensolvers which necessitate $O(N^3)$ operations to compute the eigenvectors of such ST matrices, the proposed method computes both the eigenvalues and eigenvectors with only $O(N^2)$ operations. The method is based on serial implementation of the recently introduced Divide and Conquer (DC) algorithm [3],[1],[4]. It exploits the fact that by $O(N^2)$ of DC operations, one can compute the eigenvalues of $N \times N$ ST matrix and a finite number of pairs of successive rows of its eigenvector matrix. The rest of the eigenvectors—all of them or one at the time, are computed by linear three-term recurrence relations. We conclude with numerical examples, which demonstrate the superiority of the proposed method by saving an order of magnitude in execution time at the expense of sacrificing a few orders of accuracy.

Zang, Thomas A. and M. Y. Hussaini: Numerical experiments on the stability of controlled boundary layers. ICASE Report No. 88-20, March 7, 1988, 51 pages. Submitted to J. Fluid Mechanics.

Nonlinear simulations are presented for instability and transition in parallel water boundary layers subjected to pressure gradient, suction, or heating control. In the nonlinear regime, finite amplitude two-dimensional Tollmien-Schlichting waves grow faster than is predicted by linear theory. Moreover, this discrepancy is greatest in the case of heating control. Likewise, heating control is found to be the least effective in delaying secondary instabilities of both the fundamental and subharmonic type. Flow-field details (including temperature profiles) are presented for both the uncontrolled boundary layer and the heated boundary layer.

Mehrotra, Piyush: Programming parallel architectures: The BLAZE family of languages. ICASE Report No. 88-21, March 7, 1988, 20 pages. Submitted to the Proceedings of the Third SIAM Conference on Parallel Processing for Scientific Computing.

Programming multiprocessor architectures is a critical research issue. This paper gives an overview of the various approaches to programming these architectures that are currently being explored. We argue that two of these approaches, interactive programming environments and functional parallel languages, are particularly attractive, since they remove much of the burden of exploiting parallel architectures from the user.

This paper also describes recent work by the author in the design of parallel languages. Research on languages for both shared and nonshared memory multiprocessors is described,

as well as the relations of this work to other current language research projects.

Hall, Philip and Sharon O. MacKerrell: Wave interactions in a three-dimensional attachment line boundary layer. ICASE Report No. 88-22, March 23, 1988, 34 pages. Submitted to J. Fluid Mech.

The three-dimensional boundary layer on a swept wing can support different types of hydrodynamic instability. Here attention is focused on the so-called "spanwise contamination" problem which occurs when the attachment line boundary layer on the leading edge becomes unstable to Tollmien-Schlichting waves. In order to gain insight into the interactions which are important in that problem, a simplified basic state is considered. This simplified flow corresponds to the swept attachment line boundary layer on an infinite flat plate. The basic flow here is an exact solution of the Navier-Stokes equations and its stability to two-dimensional wave propagating along the attachment line can be considered exactly at finite Reynolds number. This has been done in the linear and weakly nonlinear regimes by Hall, Malik, and Poll (1984) and Hall and Malik (1986). Here the corresponding problem is studied for oblique waves, and their interaction with two-dimensional waves is investigated. In fact oblique modes cannot be described exactly at finite Reynolds number, so it is necessary to make a high Reynolds number approximation and use triple deck theory. It is shown that there are two types of oblique wave which, if excited, cause the destabilization of the two-dimensional mode and the breakdown of the disturbed flow at a finite distance from the leading edge. Firstly, a low frequency mode closely related to the viscous stationary crossflow mode discussed by Hall (1986) and MacKerrell (1987) is a possible cause of breakdown. Secondly, a class of oblique wave with frequency comparable with that of the two-dimensional mode is another cause of breakdown. It is shown that the relative importance of the modes depends on the distance from the attachment line.

Banks, H. T. and Fumio Kojima: Boundary shape identification problems in two-dimensional domains related to thermal testing of materials. ICASE Report No. 88-23, March 30, 1988, 32 pages. Submitted to Quart. Applied Math.

This paper is concerned with the identification of the geometrical structure of the system boundary for a two-dimensional diffusion system. The domain identification problem treated here is converted into an optimization problem based on a fit-to-data criterion and theoretical convergence results for approximate identification techniques are discussed. Results of numerical experiments to demonstrate the efficacy of the theoretical ideas are reported.

ICASE COLLOQUIA

October 2, 1987 through March 31, 1988

October 2 Professor A. K. M. Fazle Hussain, University of Houston - University Park: Coherent Structures in Free-Turbulent Shear Flows October 23 Dr. John Van Rosendale, Argonne National Laboratory: Synchronized Packet Interconnection Networks November 4 Dr. Richard Barnwell, NASA Langley Research Center: A Defect Stream Function, Law of the Wall/Wake Method for Turbulent Boundary Layers November 12 **Professor Toshio Fukuda**, The Science University of Tokyo, Japan: Modeling and Control of Torsional Vibrations in a Flexible Structure November 13 Cetin Koc, University of California, Santa Barbara: Parallel Interpolation via Parallel Prefix November 16 Professor Paul Lieber, University of California, Berkeley: The Classical Navier-Stokes Theory: Dynamical Aspects of Incompleteness, Indeterminacy, and its Fundamental Augmentation November 20 Professor Michel Deville, Universite Catholique de Louvain, Belgium: Chebyshev Pseudospectral Solution of the Incompressible Navier-Stokes Equations in Rectangularly Decomposable Geometries December 10 Dr. Clive A. J. Fletcher, University of Sydney, Australia: Computational Exploitation of the Reduced Navier-Stokes Equations December 14 Dr. Burton Smith, Supercomputing Research Center: Horizon December 16 Shahid Siddiqi, Aviation Advanced Technology Applications, Orlando, Florida and AS&M, Hampton, Virginia: Trailing Vortex Rollup Computations Using the Point Vortex Method with Applications for Induced Drag Computations December 17 Professor Shin I. Aihara, Kyoto Institute of Technology, Japan: Identification of an Infinite Dimensional Parameter for Stochastic Diffusion Equations January 8 Dr. Farrokh Mistree, University of Houston - University Park: Selection and Compromise in the Conceptual Aircraft Design Professor S. Harvey Lam, Princeton University: Computational January 21 Singular Perturbation: A Mathematical Tool for Extracting Physical

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Insights

Dr. Martin Maxey, Brown University: The Settling, Dispersion, January 26 and Coagulation of Particles in Turbulence Professor Shing-Tung Yau, Harvard University: Elliptic and Heat January 28 Equations Professor Steven Orszag, Princeton University: Turbulence Sim-February 9 ulations and the Renormalization Group Theory Professor Hillel Tal-Ezer, Brown University and ICASE: On New-February 10 ton's Interpolation Formula February 23 Professor Harry Gingold, West Virginia University: More Spectral and Conformal Map Methods for Scientific Computation February 26 **Jeffrey S. Scroggs**, University of Illinois at Champaign-Urbana: Parallel Computation of a Domain Decomposition Method March 3 Dr. Tobias Orloff, University of Minnesota: Solid Modeling and Semi-Algebraic Sets March 4 Dr. Farrokh Mistree, University of Houston - University Park: Selection and Compromise in the Conceptual Aircraft Design March 18 Dr. Raymond Chin, National Science Foundation: Asymptotic-Numerical Treatment of Multiple Scale Problems

OTHER ACTIVITIES

A workshop on Control of Fluid Dynamic Systems was held March 28-29, 1988, at the Radisson Hotel, Hampton, Virginia. The purpose of this workshop was to bring together a small group of experts in fluid dynamics, applied mathematics, control sciences and computational sciences to initiate discussions on the basic research issues related to the control of fluid dynamic phenomena and to identify feasible problems and promising approaches. The following attended the workshop:

- H. T. Banks Brown University
- J. A. Burns Virginia Polytechnic Institute and State University
- P. Chow Wayne State University
- E. Cliff Virginia Polytechnic Institute and State University
- J. Crowley Air Force Office of Scientific Research
- C. Foias University of Indiana
- C. M. Ho University of Southern California
- M. Y. Hussaini ICASE, NASA Langley Research Center
- A. Jameson Princeton University
- A. Krener University of California, Davis
- L. Maestrello NASA Langley Research Center
- J. McMichael Air Force Office of Scientific Research
- W. C. Reynolds Stanford University
- R. Temam University de Paris-Sud
- L. Valvani Massachusetts Institute of Technology
- R. G. Voigt ICASE, NASA Langley Research Center

ICASE STAFF

I. ADMINISTRATIVE

Robert G. Voigt, Director Ph.D., Mathematics, University of Maryland, 1969 Numerical and Algorithms for Parallel Computers

Linda T. Johnson, Office and Financial Administrator

Etta M. Blair, Personnel/Bookkeeping Secretary

Barbara A. Cardasis, Administrative Secretary

Rosa H. Milby, Technical Publications/Summer Housing Secretary (Beginning January 1988)

Barbara R. Stewart, Technical Publications Secretary

Emily N. Todd, Executive Secretary/Visitor Coordinator

II. SCIENCE COUNCIL FOR APPLIED MATHEMATICS AND COMPUTER SCIENCE

Bruce Arden, Dean, College of Engineering and Applied Science, University of Rochester.

Andrew Callegari, Director, Theoretical & Mathematical Sciences Laboratory, Exxon Research & Engineering Company. (Through December 1987)

John Hopcroft, Joseph C. Ford Professor of Computer Science, Cornell University.

Herbert Keller, Professor, Physics, Mathematics and Astronomy, California Institute of Technology. (Through December 1987)

Stanley J. Osher, Professor, Mathematics Department, University of California.

Werner C. Rheinboldt, Andrew W. Mellon Professor, Department of Mathematics and Statistics, University of Pittsburgh.

John Rice, Head, Department of Computer Science, Purdue University.

Burton Smith, Super Computing Research Center, Institute for Defense Analysis.

Robert G. Voigt, Director, Institute for Computer Applications in Science and Engineering, NASA Langley Research Center.

III. ASSOCIATE MEMBERS

Saul S. Abarbanel, Professor, Department of Applied Mathematics, Tel-Aviv University, Israel.

H. Thomas Banks, Professor, Division of Applied Mathematics, Brown University.

David Gottlieb, Professor, Division of Applied Mathematics, Brown University.

Peter D. Lax, Professor, Courant Institute of Mathematical Sciences, New York University.

Merrell L. Patrick, Professor, Department of Computer Science, Duke University.

IV. CHIEF SCIENTIST

M. Yousuff Hussaini - Ph.D., Mechanical Engineering, University of California, 1970. Computational Fluid Dynamics. (Beginning April 1978)

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Charles G. Speziale - Ph.D., Aerospace and Mechanical Sciences, Princeton University, 1978. Fluid Dynamics with Emphasis on Turbulence Modeling and the Transition Process. (September 1987 to September 1990)

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Richard G. Carter - Ph.D., Numerical Analysis, Rice University, 1986. Numerical Methods for Optimization Problems. (September 1987 to September 1989)

Thomas W. Crockett - B.S., Mathematics, College of William and Mary, 1977. Parallel Systems Research. (February 1987 to February 1989)

Naomi Decker - Ph.D., Mathematics, University of Wisconsin-Madison, 1987. Multigrid Method Applied to Hyperbolic Equations. (September 1987 to September 1989)

David Kamowitz - Ph.D., Computer Science, University of Wisconsin, Madison, 1986. Iterative Methods for Numerical Solutions of Partial Differential Equations. (September 1986 to September 1988)

Stephen L. Keeling - Ph.D., Mathematics, University of Tennessee, 1986. Analysis of Methods for the Solution of Parabolic Partial Differential Equations. (September 1986 to September 1988)

Fumio Kojima - Ph.D., Control Theory, Kyoto University, Japan, 1985. Probabilistic and Stochastic Methods for Optimal Control Problems. (September 1986 to September 1989)

Dimitri Mavriplis - Ph.D., Mechanical and Aerospace Engineering, Princeton University, 1988 Grid Techniques for Computational Fluid Dynamics. (February 1987 to February 1989)

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- Marsha J. Berger Ph.D., Numerical Analysis, Stanford University, 1982. Research Associate, Courant Institute of Mathematical Sciences. Numerical Methods for Partial Differential Equations.
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- Max D. Gunzburger Ph.D., Mathematics, New York University, 1969. Professor, Department of Mathematics, Carnegie-Mellon University. Fluid Dynamics and Numerical Methods for Partial Differential Equations.
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- James M. Ortega Ph.D., Mathematics, Stanford University, 1962. Professor and Chairman, Department of Applied Mathematics, University of Virginia. Numerical Methods for Partial Differential equations.
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